Hanin Alrais

Quiz I: Discrete Math, MTH 213, Fall 2017

Ayman Badawi

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QUESTION 1. (i) Solve over Z, $4x \equiv 2 \pmod 6$

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$4x \equiv 2 \pmod 6$

gcd $(4, 6) = 2$

Check $2 | 2$? Yes,

we have 2 sol.

$\{6n+2,\ 6m+5\}$

$\frac{4\,\boxed{2}}{6} = 2$

$\frac{4\,\boxed{5}}{6} = 2$

$6 \overline{)8}$ = 1, -6, 2

$6 \overline{)20}$ = 3, 18, 2

(ii) Convince me that $12.45\overline{3} = 12.453333\ldots$ is rational.

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$x = 12.45333\overline{3}$

$100x = 1245.333\overline{3}$

$100x - x = \frac{123288}{100}$

$99x = \frac{123288}{100}$

$= \frac{123288}{9900}$ ← a, ← b

we can get two integers, divide them to get the value

(iii) Convert 17 to base 7

$(17)_{10}$ to base 7

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Div 7	Mod 7
17 Div 7 = 2	17 mod 7 = 3
2 Div 7 = 0 stop	2 mod 7 = 2

↑ read

$7 \overline{)17}$ = 2, -14, 3

$7 \overline{)2}$ = 0, -0, 2

$(23)_7$ check:

$2 \times 7^1 + 3 \times 7^0 = 17$

(iv) Convert $(265)_8$ to base 2

2/2

$2^{3} = 8$

$(265)_8 = (\underbrace{010}_{2}\ \underbrace{110}_{6}\ \underbrace{101}_{5})$

(v) Convert $(110110)_2$ to base 16

2/2

take 4 digits

$(\underbrace{0011}\underbrace{0110})_2 = (36)_{16}$

(vi) Given $210 \leq x \leq 420$ such that x is divisible by 10, $x \equiv 2 \ (mod\ 3)$, and $x \equiv 6 \ (mod\ 7)$. Find x.

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$X \equiv 0 \ (\text{mod } 10) \to m_1$

$X \equiv 2 \ (\text{mod } 3) \to m_2$

$X \equiv 6 \ (\text{mod } 7) \to m_3$

$\gcd(10, 3, 7) = 1$

$m = 10 \times 3 \times 7 = 210$

find $Q_2 = \frac{m}{m_2} = \frac{210}{3} \boxed{= 70}$

$y_2 = (Q_2 \text{ mod } 3)^{-1}$

$= (70 \text{ mod } 3)^{-1}$

$$\begin{array}{r} 23 \\ 3\overline{)70} \\ -69 \\ \hline 1 \end{array}$$

$1 \times \boxed{y_2} \text{ mod } 3 = 1$

$$\begin{array}{r} 0 \\ 3\overline{)1} \\ -0 \\ \hline 1 \end{array} \qquad \begin{array}{r} 1 \\ 3\overline{)4} \\ -3 \\ \hline 1 \end{array}$$

$\boxed{y_2 = 1}$ oR ~~4~~

find $Q_3 = \frac{m}{m_3} = \frac{210}{7} = 30$

$y_3 = (30 \text{ mod } 7)^{-1}$

$$\begin{array}{r} 4 \\ 7\overline{)30} \\ 28 \\ \hline 2 \end{array} \qquad (2)^{-1}$$

Check:

$$\begin{array}{r} 76 \\ 3\overline{)230} \\ 228 \\ \hline 2 \end{array}$$

$$\begin{array}{r} 32 \\ 7\overline{)230} \\ 224 \\ \hline 6 \end{array}$$

$2 \boxed{y_3} \text{ mod } 7 = 1$

$$\begin{array}{r} 1 \\ 7\overline{)8} \\ -7 \\ \hline 1 \end{array}$$

$\boxed{y_3 = 4}$

$X = \cancel{r_1 Q_1 y_1} + 70 \times 2 \times 1 + 4 \times 6 \times 30$ mod 210

$X = (140 + 720) \text{ mod } 210$

$= 860 \text{ mod } 210$

$$\begin{array}{r} 4 \\ 210\overline{)860} \\ 840 \\ \hline 20 \end{array}$$

$= 20 \text{ mod } 210$

all integers $= 210n + 20$

assume $n = 1$

$\Rightarrow 210(1) + 20 = 230$ Check:

$210 \leq 230 \leq 420$

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QUESTION 1. (i) Find $(2617)_8 - (263)_8 =$ $(2334)_8$

$$\begin{array}{r} 2\cancel{6}\cancel{1}7 \\ 263 \\ \hline 2334 \end{array}$$

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(ii) Find $(234)_5 \cdot (33)_5$

$$\begin{array}{r} 234 \\ \times \quad 33 \\ \hline 1312 \\ 1312\times \\ \hline 14432 \end{array}$$

$= (14432)_5$

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(iii) Prove $\sqrt{53}$ is irrational. [Hint: Deny. Then $\sqrt{53} = a/b$ where a, b are odd and $gcd(a, b) = 1$, now start cooking]

$\sqrt{53} = \frac{a}{b} = \frac{2m+1}{2k+1} \Rightarrow 53 = \frac{(2m+1)^2}{(2k+1)^2}$

$\Rightarrow 53(4k^2+4k+1) = 4m^2+4m+1$

$53.4k^2 + 53.4k + 53 = 4m^2+4m+1 \Rightarrow 53.4k^2 + 53.4k + 52 = 4(m^2+m)$

$\Rightarrow \underbrace{53(k^2+k)}_{\text{even}} + \underbrace{13}_{\text{odd}} = \underbrace{m^2+m}_{\text{even}}$

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LHS = odd but RHS = even $\Rightarrow$ contradiction $\Rightarrow \sqrt{53}$ is irrational

(iv) Let y be an ~~odd~~ even number. Prove that $w = y + (y+1) + (y+2)$ is ~~even~~ odd. [Hint: Direct. You must show that $w = 2k+1$ for some $k \in Z$]

let $y = 2k, k \in Z$.

$\Rightarrow w = y + (y+1) + (y+2) = 2k + (2k+1) + (2k+2) = 6k+3$

$w = 6k+3 = 2(3k+1)+1$

let $3k+1 = m, m \in Z$

$\Rightarrow w = 2m+1 \Rightarrow w$ is odd

perfect

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Name Ali M. Mandi, ID 58506

Quiz Four: Discrete Math, MTH 213, Fall 2017

Ayman Badawi

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QUESTION 1. Let $W = \{t, 6, \{t, 6\}, 3, 4, \{3\}\}$ be the universal set, $A = \{t, \{t, 6\}, 4\}$, and $B = \{6, \{3\}, t\}$

(i) Find B'

$B' = \{\{t, 6\}, 3, 4\}$

(ii) Find $A - B'$

$A - B' = \{t\}$

(iii) Find $(A \cap B)'$

$(A \cap B) = \{t\}$ $\rightarrow (A \cap B)' = \{6, \{t, 6\}, 3, 4, \{3\}\}$

QUESTION 2. a) Given $f : R \to [-2, 2]$ such that $f(x) = sin(x)$. Is $f(x)$ 1-1? Is $f(x)$ ONTO? Is $f(x)$ bijective? EXPLAIN BRIEFLY

- It is NOT ONTO, because 2 & -2 are not used in the Co-domain.
- It is NOT 1-1, because there are elements in the domain that Points to the same element in the Co-domain.
  ex: $\frac{\pi}{2}, 0 \longrightarrow \sin(\frac{\pi}{2}) = 0$, $\sin(0) = 0$

- Not bijective. because it is not 1-1 nor ONTO.

b) Given $f : [0, \infty) \to [1, \infty)$ such that $f(x) = \sqrt{x} + 1$. Is $f(x)$ 1-1? Is $f(x)$ ONTO? Is $f(x)$ bijective? EXPLAIN BRIEFLY

- It's 1-1, because every element in the domain gives you only one element in the Co-domain.
- It's ONTO because every element in the domain Points to an element in the Co-domain and no elements are left

- It is bijective because it is 1-1 & ONTO.

c) Given $f : (4, \infty) \to (-3, \infty)$ such that $f(x) = \frac{1}{x-4}$.

- It is 1-1, because every element in the domain gives you one element in the Co-domain.
- It's Not ONTO because some elements in the Co-domain are not being used.

- It's NOT Bijective.

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Name Hanin AlKais, ID 66229

Quiz 6: MTH 213, Fall 2017

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QUESTION 1. Given string1, say S_1 : 1 0 1 0 1 1 and string2, say S_2 : 0 1 1 1 0 1

100010

a) Find $S_1 \wedge \neg S_2$

100010

b) Find $S_1 \vee S_2$

111111

c) Find $S_1 \oplus S_2$

110110

QUESTION 2. Convince me that $S_1 \wedge (S_1 \vee S_2) \equiv S_1 \vee (S_1 \wedge S_2)$ Truth table. identical truth table

S_1	S_2	$S_1 \vee S_2$	$S_1 \wedge (S_1 \vee S_2)$	$S_1 \wedge S_2$	$S_1 \vee (S_1 \wedge S_2)$
0	0	0	0	0	0
0	1	1	0	0	0
1	0	1	1	0	1
1	1	1	1	1	1

QUESTION 3. Given a sequence $\{b_n\}_{n=0}^{\infty}$, where $b_0 = 5, b_1 = 1$, and $b_n = -3b_{n-1} + 10b_{n-2} + \sqrt{n} + 3$. Find a general formula for b_n.

$$b_n = -3b_{n-1} + 10b_{n-2} + \sqrt{n} + 3$$

$$\frac{\alpha^n}{\alpha^{n-2}} = -\frac{3\alpha^{n-1}}{\alpha^{n-2}} + \frac{10\alpha^{n-2}}{\alpha^{n-2}}$$

$$\alpha^2 + 3\alpha - 10 = 0 \Rightarrow (\alpha+5)(\alpha-2) = 0$$

$$\alpha = 2, \quad \alpha = -5$$

$$b_n = C_1(2)^n + C_2(-5)^n + \sqrt{n} + 3$$

$$b_0 = 5 = C_1 + C_2 + 0 + 3 \Rightarrow C_1 + C_2 = 2 \quad \text{— (1)}$$

$$b_1 = 1 = 2C_1 - 5C_2 + 1 + 3$$

Faculty information $\Rightarrow 2C_1 - 5C_2 = -3$ — (2)

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$$C_1 = 1, \quad C_2 = 1$$

gen. formula : $b_n = (2)^n + (-5)^n + \sqrt{n} + 3$

Quiz 7: MTH 213, Fall 2017

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Domain : persons   Range : day of week

1 ... 47 $\xrightarrow{\text{onto}}$ 1 ... 7

|Range| < |Domain|

QUESTION 1. 1) Assume there are 47 persons in a class room. Then there are at least n persons who were born on the same day of the week. What is the maximum value of n that we all are sure about?

By Pigeon hole Principle: There are at least $\left\lceil \frac{47}{7} \right\rceil$ persons born on the same day of week

max. $n = 7$

1/1

2) Assume that a class has 8 males and 4 females. We need to form a committee with 3 persons.

(i) In how many ways can we form a committee with at least 2 males are selected?

$= 12C3 - (8C0 \times 4C3 + 8C1 \times 4C2)$

1/2

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(ii) In how many ways can we form a committee with at least 1 female is selected?

$= 12C3 - (8C3 \times 4C0)$

1/2

(iii) In how many ways can we form a committee with 1 male and 2 females?

$= 8C1 \times 4C2$

1/2

QUESTION 2. Consider the following Algorithm segment. Find the exact number of additions, multiplications, and subtractions that will be performed when the algorithm is executed. Then find the order of the Algorithm segment.

$m = 7$

For $k := 3$ *to* $n + 4$

For $i := 1$ *to* k

$s = m^2 + 3 * i - i * k$ → 3 multiplications; 1 addition & 1 subtraction. Total operations = 5

next i

$m = m * k + m$ → extra 1 multiplication & 1 addition = 2 operations when outer loop is executed.

next k

Outer loop is iterated $n+4-3+1 = \boxed{n+2}$ times.

for a given value of k; inner loop is iterated $k-1+1 = \boxed{k}$ times.

when $k = 3$; inner loop is iterated 3 times } difference is always 1

$k = 4$; inner loop is iterated 4 times.

when $k = n+4$; inner loop is iterated $\boxed{n+4}$ times.

This is an arithmetic sequence: sum $= \frac{(n+2)(3+n+4)}{2} = \frac{(n+2)(7+n)}{2}$

Total number of operations $= \frac{5}{2}(n+2)(7+n) + \underbrace{2(n+2)}_{\text{extra when outer loop is executed}} =$

$= 5/2\,(7n + n^2 + 14 + 2n) + 2n + 4 = 2.5n^2 + 22.5n + 35 + 2n + 4$

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Total number of operations $= \boxed{2.5n^2 + 24.5n + 39}$

This algorithm segment is of order $\Theta(n^2)$ and it is also of order $O(n^2)$